

N 9 4 - 1 5 5 8 5

## NEW PERSPECTIVES FOR HIGH ACCURACY SLR WITH SECOND GENERATION GEODESIC SATELLITES

Glenn LUND

*Optical Department*

*AEROSPATIALE - Space & Defense Division*

*100 Boulevard du Midi*

*06322 CANNES LA BOCCA*

*FRANCE*

### ABSTRACT

This paper reports on the accuracy limitations imposed by geodesic satellite signatures, and on the potential for achieving millimetric performances by means of alternative satellite concepts and an optimised 2-colour system tradeoff.

Long distance laser ranging, when performed between a ground (emitter/receiver) station and a distant geodesic satellite, is now reputed to enable short arc trajectory determinations to be achieved with an accuracy of 1 to 2 centimeters. This state-of-the-art accuracy is limited principally by the uncertainties inherent to single-colour atmospheric pathlength correction. Motivated by the study of phenomena such as post-glacial rebound, and the detailed analysis of small-scale volcanic and strain deformations, the drive towards millimetric accuracies will inevitably be felt.

With the advent of short pulse ( $< 50$  ps) dual wavelength ranging, combined with adequate detection equipment (such as a fast-scanning streak camera or ultra-fast solid-state detectors) the atmospheric uncertainty could potentially be reduced to the level of a few millimeters, thus exposing other less significant error contributions, of which by far the most significant will then be the morphology of the retroreflector satellites themselves.

Existing geodesic satellites are simply dense spheres, several 10's of cm in diameter, encrusted with a large number (426 in the case of LAGEOS) of small cube-corner reflectors. A single incident pulse thus results in a significant number of randomly phased, *quasi*-simultaneous return pulses. These combine coherently at the receiver to produce a convolved interference waveform which cannot, on a shot to shot basis, be accurately and unambiguously correlated to the satellite center of mass.

The present paper proposes alternative geodesic satellite concepts, based on the use of a very small number of cube-corner retroreflectors, in which the above difficulties are eliminated whilst ensuring, for a given emitted pulse, the return of a single clean pulse with an adequate cross-section.

## 1 RECENT AND IMMINENT ADVANCES IN SLR ACCURACY

Many SLR groups have reported, during the course of the present workshop, considerable progress in their laser station equipment, as well as in the performance of data reduction and orbital modeling.

Laser technology has made considerable progress since the days of the first ruby lasers, and most stations are now equipped with a doubled Nd:YAG. Pulse widths down to 50 ps, with an output power of ~ 30 mJ and a repeat rate of ~ 10 Hz are now being reported and can be expected to become routine performances in many stations. The temporal width of such pulses is now responsible for much less of the return signal duration than the optical depth of the satellite itself.

The technology of ultra-fast SPAD (Silicon Photo Avalanche Diode) detectors would seem very promising for picosecond event timing. Streak cameras, despite the considerable expense involved in their exploitation, are recognised not only for their single picosecond capacity but also for their ability to record single or multiple waveforms (eg. for 2-colour differential flight time measurements). K. Hamal has reported streak camera satellite signature recordings from AJISAI and STARLETTE.

The accuracy of atmospheric models, used to correct for the delay in pulse flight time, is considered to be limited by the small amount of meteorological data exploited and from which the entire atmospheric profile is implicitly defined. Although single-colour SLR makes use of a single *in situ* measurement of temperature pressure and water vapour pressure, 2-colour ranging is expected to enable a considerable improvement to be achieved by virtue of differential flight time correction. The residual error with this technique lies in the range of 2 to 3 mm.

Although several authors have reported improvements in the modeling used to take into account both gravitational and non-gravitational (radiation, thermal, drag) orbital influences, the residual bias in the determination of LAGEOS' orbit is estimated as 2 to 3 cm for 1 month of data and 1 cm for a year of data (R. Eanes). Many sophisticated data reduction and collation procedures have been elaborated, but the limit in rms baseline accuracy nevertheless appears to saturate at the level of 1 cm.

## 2 RETURN SIGNAL CONVOLUTION BY MULTIPLE ECHOS

The time domain behaviour of a round-trip SLR pulse can be thought of as an initially narrow Gaussian pulse which is convolved successively by the atmospheric transfer function (due to refractive index dispersion - typically ~ 10 ps), the "forest" of Diracs corresponding to each of the satellite reflectors "visible" at the ranging instant, the atmospheric transfer function for a second time, and finally the detector transfer function. Whereas the emission, detection and atmospheric functions could well provide a global received pulse FWHM of ~ 50 ps (7.5 mm), the satellite signature, determined by the overlapping reflector echos within its apparent optical depth, is typically an order of magnitude greater in duration.

The practically indeterminant and undeconvolvable nature of the resulting signature imposes a practical limit in residual range uncertainty varying between 1 and 5 cm (depending on the size and characteristics of the ranged satellite), even after several months of data accumulation. This uncertainty is represented by  $\sigma^2_{\text{sat}}$  (~  $\geq 100$  ps) in expression (1) of §3.

In an effort to eliminate the drawbacks of such complex signatures, a novel geodesic satellite concept is proposed in which a very small number of retroreflectors, exhibiting very little or no local FOV overlap, is arranged in such a way as to ensure the unicity (or at least unambiguous identifiability) of the detected return waveforms. Ideally, the apexes of each retroreflecting cube corner would be mutually co-located at the satellite's centre of mass. In practice, this requirement cannot be exactly met, implying either some form of attitude stabilisation and/or deterministic correction between measured and true (centre of mass) ranges. The residual uncertainty in range correction should be no more than 1 or 2 mm.

Velocity aberration is a parameter of considerable importance, and the means used to achieve appropriate correction would depend on the implementation (or not) of attitude stabilisation.

Finally, an adequate systems approach to high accuracy SLR would require a careful tradeoff to be made between the mean satellite reflector cross-section (i.e. related to the cube corner size, mean incidence angle and diffraction lobe pattern) and station parameters such as (dual colour) laser wavelength choice, pulse energy and width, and receiving telescope size. The latter considerations are addressed on the following section.

### 3 LINK BUDGET REQUIREMENTS AND SYSTEM CONSIDERATIONS

The parameters which exert an influence on range accuracy include not only the retroreflector characteristics, but also the laser station design. Range accuracy and link budget analyses are thus needed in order to ascertain an optimal approach to the specification of high accuracy SLR satellites and 2-colour ranging stations, as set out below. The uncertainty in the absolute value of a 2-colour corrected normal point can be expressed as;

$$\sigma_{norm.pt.}^2 = K_1 \left\{ \sigma_{sat}^2 + \frac{\sigma_{noise}^2}{N_{shot}} + A^2 \left( \sigma_{sc}^2 + \frac{1}{N_{shot}} \left( \frac{\sigma_{pulse}^2}{N_{det}(\lambda_1)} + \frac{\sigma_{pulse}^2}{N_{det}(\lambda_2)} \right) \right) \right\} \quad (1)$$

where ;

- $\sigma_{sat}^2$  is the unaccountable satellite center-of-mass bias,
- $\sigma_{noise}^2$  is the cumulated random variance of the timing system,
- $N_{shot}$  is the number of shots used to establish a normal point,
- $K_1$  is a conversion factor from ps to mm,
- $\sigma_{sc}^2$  is the unaccountable Streak Camera bias (typically 1 to 2 ps),
- $\sigma_{pulse}^2$  defines the received pulse width ( $\sigma = 0.425$  FWHM),
- $A$  is the 2-colour correction sensitivity (related to  $\lambda_1$  and  $\lambda_2$ ), and
- $N_{det}(\lambda)$  is the number of detected photons at wavelength  $\lambda$ .

The expected number of detected photons at a given wavelength is derived from the link budget analysis as follows :

$$N_{det} = K_2 \left\{ \lambda \cdot \frac{1}{R^4(z)} \cdot \frac{1}{\omega^2} \cdot (1 - \theta/40) \cdot T_{atm}^2(z) \cdot QE(\lambda) \right\} \times \left\{ \frac{E_{emit} \cdot \Phi_{tel}^2 \cdot \Phi_{cc}^2}{\Omega^2} \right\} \quad (2)$$

where ;

- $K_2$  is a constant including geometrical factors optical efficiencies,
- $R$  is the station to satellite range,
- $\omega$  is the emitted beam divergence,
- $\theta$  is the local incidence angle of the ranging beam relative to the reflector normal.  $(1 - \theta/40)$  gives a close approximation to the cube-corner cross-section falloff as a function of incidence angle),
- $T_{atm}$  is the 1-way atmospheric transmission, depending on range angle  $Z$ ,
- $QE_\lambda$  is the receiver detector quantum efficiency at wavelength  $\lambda$ ,
- $E_{emit}$  is the emitted pulse energy,
- $\Phi_{tel}$  is the station receiving telescope diameter,
- $\Phi_{cc}$  is the retroreflector diameter, and
- $\Omega$  is the equivalent angular diameter of the retroreflector diffraction pattern.

Expression (2) has been grouped into two parts of which the first includes non-system or typically invariant parameters, whereas the second contains those variables which depend on the station (laser, telescope) and satellite (retroreflector size, diffraction pattern) designs.

The term  $\sigma_{pulse}$  in (1) is also considered as a system parameter, and has been combined with the second group of variables in (2) to form the quantity  $X_{syst}$  :

$$X_{syst: \sigma, E, \Phi_{tel}, \Phi_{cc}, \Omega} = \frac{E_{emit} \cdot \Phi_{tel}^2 \cdot \Phi_{cc}^2}{\Omega^2 \cdot \sigma_{pulse}^2} \quad (J.m^4.rad^{-2}.s^{-2}) \quad (3)$$

Tradeoff analysis of the satellite and laser station characteristics can be made on the basis of the numerical evaluation of this quantity. Although the factor  $A$  is also a system-related parameter, its numerical value happens to be very similar for the 2 cases treated below and is therefore not included in the above expression. In the following, it is assumed that  $\Omega$  is not dependant on  $\Phi_{cc}$ , but rather on an engineered diffraction lobe pattern; a conservatively large value of  $\Omega$  is taken, corresponding to diffraction into a uniform ring 2.5 arcsecs in width, at a mean distance of 9 arcsecs from the incoming beam direction.

Figures (1) and (2) thus illustrate the computed variation of the 10-shot ranging accuracy  $\sigma_{Norm.Pt.}$  (exp. 1) as a function of  $X_{syst.}$ , for a 800 km circular orbit and negligible center-of-mass bias ( $\sigma_{sat}$ ), at four values of zenith ranging angle :  $0^\circ$ ,  $20^\circ$ ,  $40^\circ$  &  $60^\circ$  (assuming 10 km Standard Atmosphere visibility), and for two different cases of dual wavelength laser configuration :

Fig.1 Doubled & tripled Nd:YAG,

Fig.2 Fundamental & doubled (eg. Ti-Sapphire) wavelengths @ 800 & 400 nm.

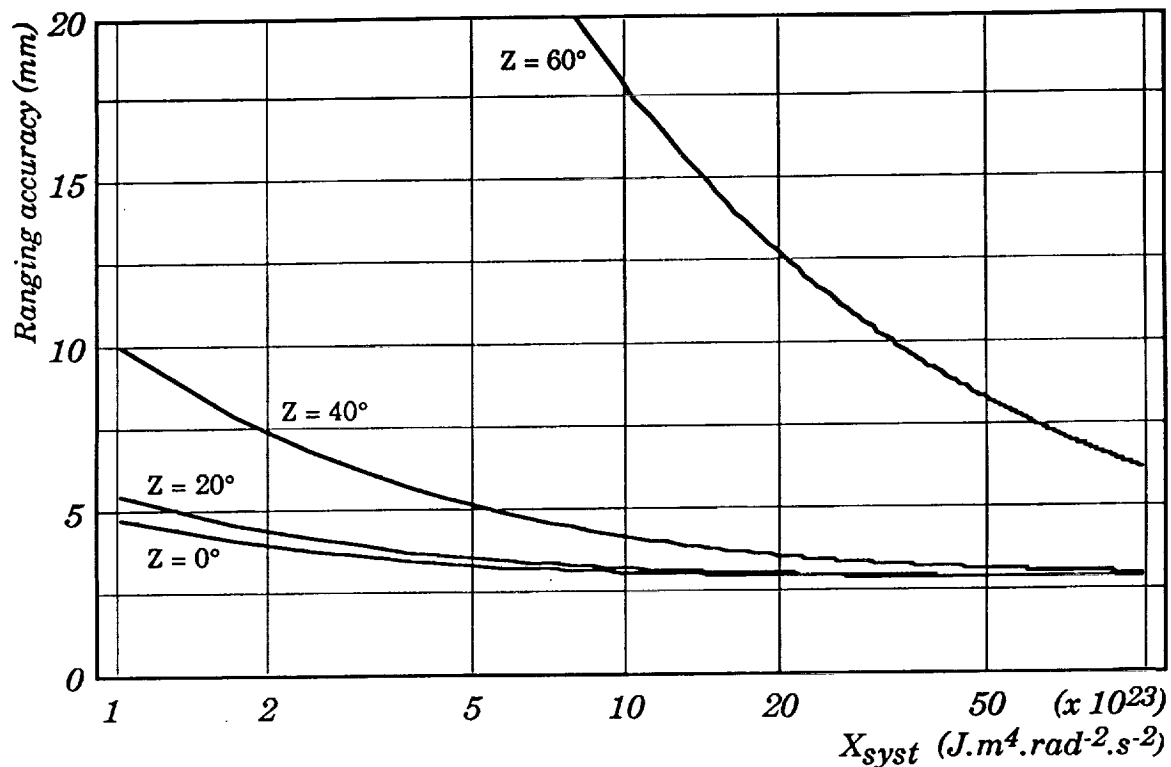


Figure 1. Nd:YAG 2-colour ranging accuracy as a function of the system input parameter  $X (= E.\Phi_{tel}^2.\Phi_{cc}^2 / \Omega^2.\sigma_{pulse}^2)$  and zenith ranging angle  $Z$ .

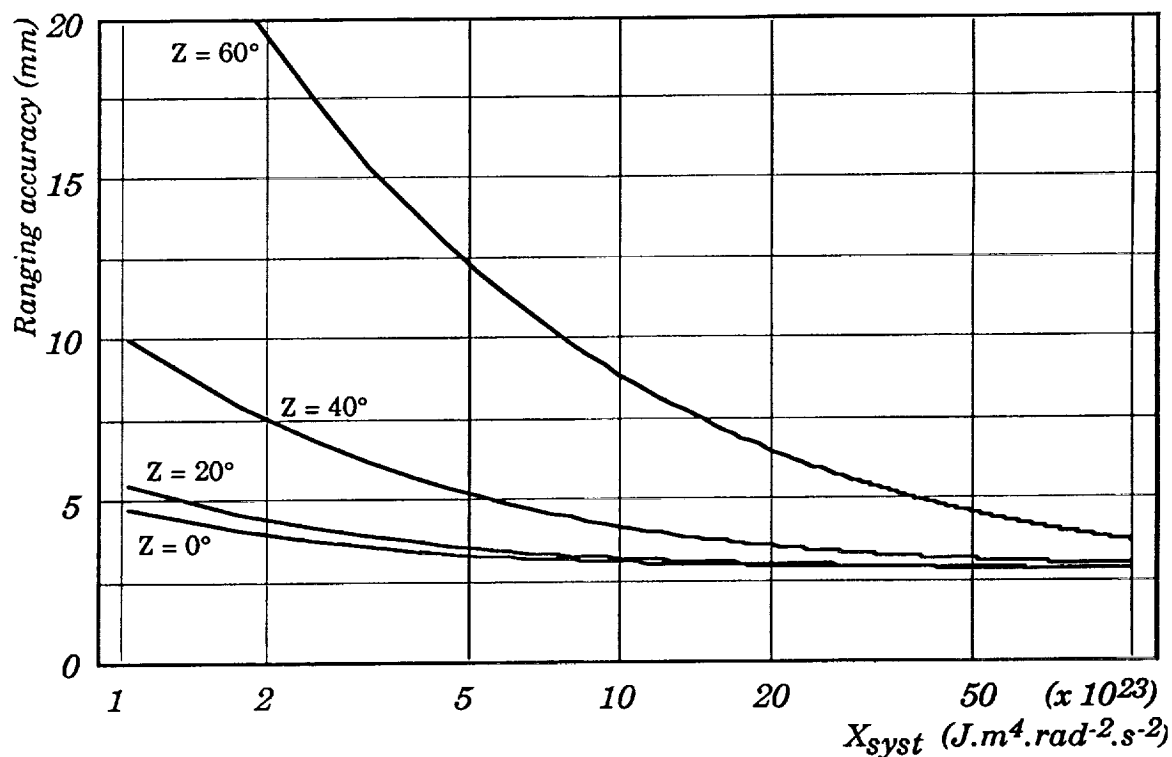


Figure 2. 400 / 800 nm 2-colour ranging accuracy as a function of the system input parameter  $X (= E.\Phi_{tel}^2.\Phi_{cc}^2 / \Omega^2.\sigma_{pulse}^2)$  and zenith ranging angle  $Z$ .

The resulting curves show (for example) that if sub-centimetric ranging accuracies are to be achieved with 2-colour systems up to zenith angles of  $60^\circ$ , one could accept a value of  $X_{\text{sys}} \geq 7.10^{23}$  from the fundamental/doubled system, whereas a factor of  $\sim 8$  improvement would be needed for comparable performance from a Nd:YAG station.

The above example supposes *normal* incidence at the retroreflector, and could be provided by the following numerical values for a 800 / 400 nm system and a *single* cube-corner retroreflector :

- $E_{\text{emit}} = 40 \text{ mJ}$
- $\Phi_{\text{tel}} = 30 \text{ cm}$
- $\Phi_{\text{cc}} = 4 \text{ cm}$
- $\sigma_{\text{pulse}} = 43 \text{ ps}$  ( $\text{FWHM}_{\text{pulse}} = 100 \text{ ps}$ )

By shortening the emitted pulse FWHM to  $\sim 50 \text{ ps}$ , one could achieve the same performance at local retroreflector incidences up to  $30^\circ$ . An additional factor of  $\sim 3$  improvement in telescope or reflector diameter would be required to achieve the same result with a Nd:YAG laser (assuming 40 mJ and 50 ps FWHM to be near to the practical limits for the emitted pulse power).

Further improvements could be achieved with still shorter pulses (in the case of a broad bandwidth amplification medium), or a larger number of pulses per normal-point. At the satellite level, larger retroreflectors, or more confined diffraction lobe patterns could be considered, without having to resort to the classical multiple reflector solution.

The strong link budget dependance on orbital height ( $\propto R^{-4}$ ) would probably render single-reflector satellites unsatisfactory for orbits higher than a few 1000 km, if sub-centimetric accuracies were to be expected at high zenith angles and with a small number of pulses per normal point.

These considerations are encouraging, as they suggest that millimetric center-of-mass determinations could well be achieved with dual-colour SLR, provided appropriate consideration is given to the design of future geodesic satellites.

#### 4 SECOND GENERATION REFLECTOR SATELLITE CONFIGURATIONS

It has been pointed out at the beginning of this paper that in the interests of a clean return signature and unambiguous center-of-mass determinations, the "ideal" geodesic satellite should exhibit non-overlapping reflector FOVs in order to inhibit multiple echo returns, whereas the link budget calculations given in § 3 show that a *single* retroreflector of reasonable dimensions could provide adequate return signal strength for millimetric range determination accuracies.

A parameter of considerable importance in the retroreflector design is that of velocity aberration correction, in which an appropriate trade-off is needed between link budget considerations (influenced by the value of  $\Omega$ ) and stabilisation of the reflector orientation relative to the orbital plane. Two fundamentally different approaches could thus be used, according to the choice between ultimate satellite simplicity and optimal link budget performance, as set out in the examples below :

#### 4.1 UNSTABILISED OMNIDIRECTIONAL RETROREFLECTOR SATELLITE

An *omnidirectional* diffraction pattern, as used in the example in § 3 and in the case of existing geodesic satellites, relieves the spacecraft from any attitude stabilisation requirements, and thus results in the simplest form of satellite configuration. In the example shown in figure (3), the satellite is composed of 8 cube corner retroreflectors (solid or hollow, depending on thermal, optical and dimensional constraints), each designed to provide an annular diffraction lobe.

If the ranging system and satellite were designed so as to meet the link budget requirements for local incidence angles up to  $30^\circ$ , it can be shown that any emitted pulse would have a 55% probability of returning to the receiver with adequate signal strength. When compared with the potential advantage of highly accurate range determinations, the drawback of intermittently weak return signals does not appear to be of significant concern.

Although the finite thickness of the cube corner walls would lead to a small residual uncertainty in the satellite center-of-mass determination, this effect is found to be negligible for wall thicknesses up to  $\sim 2$  cm.

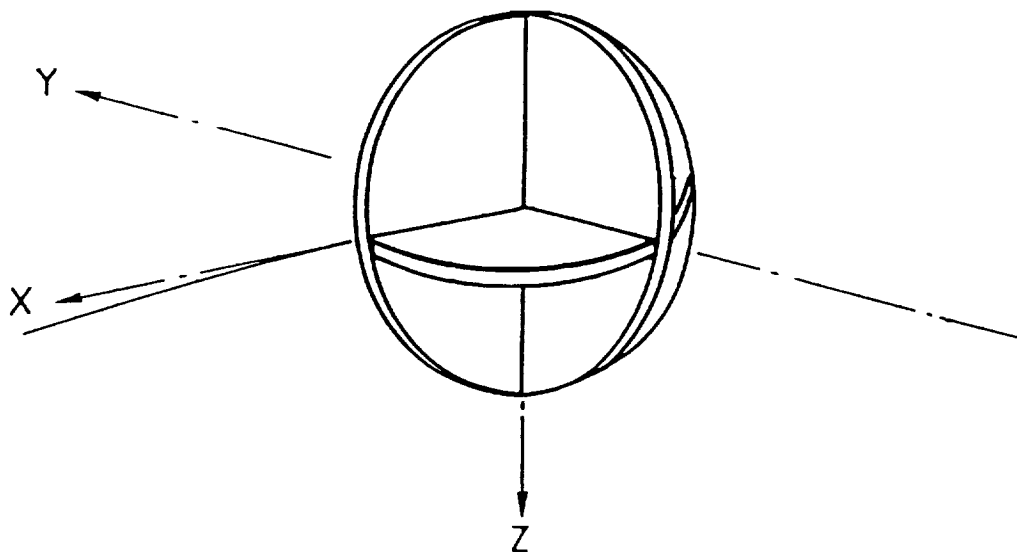


Figure 3 Passive geodesic satellite design for millimetric range determinations.

## 4.2 STABILISED DIRECTIONAL RETROREFLECTOR SATELLITE

The link budget performance of the previous design could be considerably improved by concentrating the diffracted energy into 2 small lobes (i.e. by reducing  $\Omega$ ), as is the case with the proposed GLRS-R ground target design (reported elsewhere in these proceedings). This approach could only be implemented if the direction of the diffraction lobes were appropriately maintained in alignment with the direction of relative movement between emitter and reflector.

In Figure (4), such a satellite design is shown, in which spin stabilised attitude control is used to maintain one of each of the cube corner dihedral angles in a direction perpendicular to the orbital plane. These dihedral angles would be slightly spoiled from perpendicularity so as to generate two small diffraction lobes in opposite directions, of sufficient amplitude to correct for the velocity aberration. A gain of  $\sim 10$  or more in return signal strength, compared with the annular diffraction lobe design, could be expected.

Although a spin stabilisation is shown here, 3-axis stabilisation could also be envisaged - thus requiring a smaller number of cube corners. In both cases the attitude control would not have to be very precise (typically  $\pm 5^\circ$  should be adequate), and could be achieved using virtually passive systems such as magnetic torquers. A further possibility would be to use a completely passive 3-axis stabilisation system based on gravity masts.

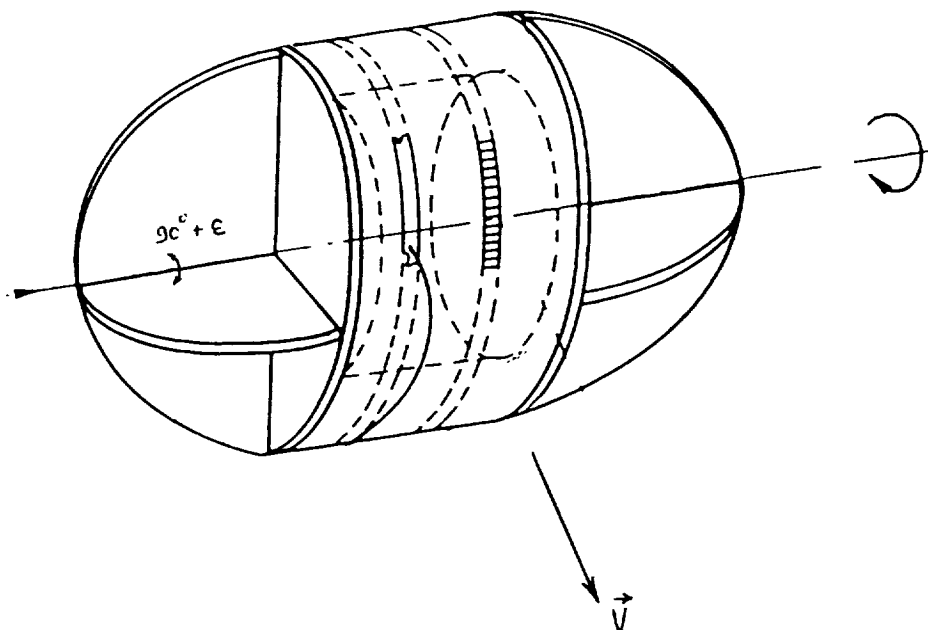


Figure 4 Spin-stabilised geodesic satellite design for millimetric range determinations.



## 5 CONCLUSIONS

It has been shown from link budget analyses that, when coupled with an appropriate satellite configuration, realistic *single* retroreflector dimensions could enable millimetric absolute accuracies to be achieved with future 2-colour SLR, through the removal of center-of-mass uncertainties in the range determination.

The engineering of retroreflector velocity aberration corrections is an important factor in the design of a geodesic satellite and in the link budget performance to be expected from the complete ranging system.

Two approaches to the design of second generation geodesic satellites have been suggested. Although these might imply the use of non-spherical orbiting bodies, with consequently higher atmospheric drag and solar radiation pressure imbalances than for existing satellites, they could nevertheless enable excellent geometric mode (short arc) range determinations to be achieved, and could be expected to provide very good long arc performance for high altitude orbits.

**SLR Data**

**Analysis/Model Errors**